Extra Dimensions and Physics of Low-Scale Strings

额外维度与低能弦物理

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This review aims to provide a very short and pedestrian introduction to some of the basics of extra-dimensional physics. The hope is to facilitate access and to be, in some respects, complementary to the many already existing reviews on phenomenological applications of extra dimensions in our Universe.

本综述旨在非常浅显简要地介绍额外维度物理的部分基础内容，希望能帮助读者入门，并且就部分内容而言，补充现有众多关于额外维度在我们宇宙中唯象应用的综述。

Keywords

关键词

Extradimensions - Gravity - String theory

额外维度 - 引力 - 弦论

Introduction

引言

Is there any compelling reason to believe that there are extra dimensions? If so, why haven't we seen them? Could we discover them in the near future? How would they manifest themselves? These are the questions we face, and this review, which is intended to be a very basic introduction to the subject, is never intended to be exhaustive in any sense. We attempt to give an overview of some of the issues studied, which we hope will serve as a starting point for the nonexpert reader to explore the vast literature on the subject.

是否有令人信服的理由相信额外维度存在？如果存在，我们为什么至今没有观测到它们？我们能在不久的将来发现它们吗？它们会以什么形式呈现？这些就是我们要面对的问题，本篇综述是该领域非常基础的入门介绍，绝非旨在穷尽所有相关内容。我们尝试对部分已研究的问题给出概述，希望能帮助非专业读者以此为起点，探索该领域的大量文献。

Historically, the possibility of an extra dimension in physics is motivated by the desire to write a theory that unifies gravity with other interactions. This was the idea behind the early work on the subject by Kaluza [1], Klein [2, 3], Einstein, and Bergmann [4] in particular at the beginning of the twentieth century (for a discussion on some aspects the history of the subject, see, e.g., [5]). Remarkably, their ideas are central, decades later, to the modern approach to the quantification of gravity: string theory. By moving from the notion of fundamental point particles to zero modes of string oscillations, the consistency of our quantum theory then requires us to move to a dimension of space-time greater than four. Indeed, string theory, which offers a framework to unify all interactions, requires the existence of additional degrees of freedom which, in certain limits, take the form of additional dimensions. Let us note here that these dimensions are space and nontemporal in nature, the latter posing problems with the preservation of causality. The possibility of observing extra dimensions has been proposed in several works [6-10].

从历史上看，物理学中额外维度的可能性，源于人们想要构建一个能够将引力与其他相互作用统一起来的理论。这正是 20 世纪初卡鲁扎 [1]、克莱因 [2,3]、爱因斯坦与伯格曼 [4] 等人早期相关研究的核心思想 (关于该领域历史的部分探讨，可参见例如 [5])。值得注意的是，数十年后，他们的思想仍是现代量子引力研究方法——弦理论的核心。弦理论将基本点粒子的概念替换为弦振荡的零模，为了保证量子理论的自洽性，我们需要将时空维度拓展到四维以上。事实上，提供了所有相互作用统一框架的弦理论，要求存在额外的自由度，在特定极限下这些自由度就表现为额外维度。在此需要说明，这些维度本质上都是空间维度，而非时间维度——时间额外维度会给因果性守恒带来问题。已有多项研究 [6-10] 提出了观测额外维度的可能性。

Obviously, in a first step, it is natural to start by characterizing the additional dimensions by their number, 1, 2, 3, etc., their topology, their geometry (flat, curved, wrapped, fractal), and for the physical phenomena we are interested in, the type of states that propagate and interact in them. In general, one is then led to discuss physics in a plethora of spaces of dimensions higher than four, with a huge choice of possible topologies and geometries. We will concentrate here on the simplest configurations that allow us to illustrate some basic examples.

显然，研究的第一步自然是从刻画额外维度的基本性质开始: 它们的数目 (1 个、2 个、3 个……)、拓扑、几何 (平坦、弯曲、卷曲、分形)，以及对我们关注的物理现象而言，在其中传播和相互作用的粒子态的类型。一般来说，我们因此需要讨论大量四维以上空间中的物理，可能的拓扑和几何选择非常多。本文我们将集中讨论最简单的构型，以此举例说明一些基础概念。

The review is organized as follows: section ”Kaluza-Klein Excitations” presents Kaluza-Klein states. While this is done in the simplest setting of a compactification on a circle, we discuss properties that remain valid in a much larger setting.

本篇综述结构安排如下: “卡鲁扎-克莱因激发”一节介绍卡鲁 za-克莱因态。虽然我们是在圆紧致化这一最简单的框架下展开讨论，但我们讨论的相关性质在更广泛的场景中依然成立。

Section ”Searches for Extra Dimensions” discusses the current experimental limits on the existence of such dimensions. Some applications of these dimensions, relevant for the construction of extensions beyond the Standard Model, are discussed in the rest of the review.

“额外维度搜寻”一节讨论当前对这类维度存在性的实验限制。在综述的其余部分，我们会讨论这些维度在构建超出标准模型的扩展理论方面的一些相关应用。

Kaluza-Klein Excitations

卡鲁扎-克莱因激发

We need to introduce some basic notions of physics with extra dimensions useful for our discussions. For this purpose, we shall start by reviewing the simplest model of dimensional reduction from $D+1$ to D dimensions (We work with the signature $(+,-,\ldots,-)$. The $D+1$ dimensional quantities will be denoted with a hat. We use Latin and Greek letters for the $D+1$ and D -dimensional coordinates; $\hat{g}$ and g are the determinants of the $D+1$ and D -dimensional metric, respectively.). We denote the coordinates of the D non-compact directions by x^m . The $(D+1)$ -th dimension is parametrized by the coordinate $z \equiv z + 2\pi R$ taken to be a circle of radius R . We consider the Einstein-Hilbert action:

> 我们需要介绍一些对后续讨论有用的额外维度基础物理概念。为此，我们先回顾从 $D+1$ 维到 D 维维度约化的最简单模型 (我们采用号差 $(+,-,\ldots,-)$ ，$D+1$ 维物理量加上标帽，用拉丁字母和希腊字母分别标记 $D+1$ 维和 D 维坐标；$\hat{g}$ 和 g 分别是 $D+1$ 维和 D 维度规的行列式)。我们将 D 个非紧致方向的坐标记为 x^m ，第 $(D+1)$ 维用坐标 $z \equiv z + 2\pi R$ 参数化，该维度是半径为 R 的圆。我们考虑爱因斯坦-希尔伯特作用量:

$$\mathcal{S}_{EH}^{(D+1)} = \frac{1}{2\hat{\kappa}^2} \int \mathrm{d}^{D+1}x \sqrt{(-1)^D \hat{g}} \widehat{\mathcal{R}}, \tag{1}$$

where $\widehat{\mathcal{R}}$ is the Ricci scalar. We take for the metric $\hat{g}_{MN}$ the ansatz:

> 其中 $\widehat{\mathcal{R}}$ 是里奇标量，我们对度规 $\hat{g}_{MN}$ 取如下拟设:

$$\hat{g}_{MN} = \begin{pmatrix} e^{2a\phi} g_{\mu\nu} - e^{2b\phi} A_\mu A_\nu & e^{2b\phi} A_\mu \\ e^{2b\phi} A_\nu & -e^{2b\phi} \end{pmatrix} \tag{2}$$

where ϕ is the radion/dilaton, A_μ the graviphoton, $g_{\mu\nu}$ the D -dimensional metric, and a, b are constants to be determined in the following. It is easy to see that the invariance under general coordinate transformations of g_{MN} becomes gauge invariance of A_μ .

> 其中 ϕ 是半径子/伸缩子，A_μ 是引力光子，$g_{\mu\nu}$ 是 D 维度规，a, b 是后续需要确定的常数。不难看出，g_{MN} 广义坐标变换的不变性对应 A_μ 的规范不变性。

Indeed, the general coordinate transformations of the $(D+1)$ -dimensional theory, depending on the vector parameter ξ^M , act on the metric as

> 事实上，$(D+1)$ 维理论的广义坐标变换依赖于矢量参数 ξ^M ，它对度规的作用为

$$\delta_\xi \hat{g}_{MN} = \xi^P \partial_P \hat{g}_{MN} + 2\partial_{(M} \xi^P \hat{g}_{N)P}, \tag{3}$$

where pairs of indices in parenthesis are symmetrized with a multiplicative 1/2- factor. It follows that the field ϕ transforms under the $(\xi^\mu = 0, \xi^z)$ transformations as

> 括号内的指标对按乘以因子 1/2 对称化，由此可得场 ϕ 在 $(\xi^\mu = 0, \xi^z)$ 变换下的变换形式为

$$\delta_{\xi^z} \phi = b \partial_z \xi^z - \xi^z \partial_z \phi. \tag{4}$$

This implies that we can gauge-fix the transformations ξ^z for $\partial_z \xi^z \neq 0$ by taking ϕ to be independent of z :

这意味着我们可以对 $\partial_z \xi^z \neq 0$ 的变换 ξ^z 做规范固定，要求 ϕ 不依赖于 z :

$$\partial_z \phi = 0 \, . \tag{5}$$

Similarly, we consider the ξ^v transformations of the off-diagonal metric components:

类似地，我们考虑非对角度规分量的 ξ^v 变换:

$$\delta_{\xi^v} \hat{g}_{\mu z} = \xi^v \partial_v \hat{g}_{\mu z} + \hat{g}_{vz} \partial_\mu \xi^v + \hat{g}_{\mu v} \partial_z \xi^v . \tag{6}$$

It follows that we can also gauge-fix the ξ^v transformations for $\partial_z \xi^v \neq 0$ by taking the off-diagonal components of the metric to be z -independent:

由此我们也可以对 $\partial_z \xi^v \neq 0$ 的 ξ^v 变换做规范固定，要求度规的非对角分量不依赖于 z :

$$\partial_z \hat{g}_{\mu z} = 0. \tag{7}$$

As a result of the above gauge conditions, in the metric decomposition (2), only the D -dimensional metric $g_{\mu\nu}(x^m, z)$ depends on all the $(D+1)$ coordinates, while the radion $\phi(x)$ and graviphoton $A_\mu(x)$ are independent of the compact coordinate z . Moreover, the leftover coordinate transformations depend only of the non-compact coordinates x^m : $\xi^\mu(x)$ generate the D -dimensional coordinate transformations, while $\xi^z(x)$ acts as a $U(1)$ gauge transformation on the gravi-photon $A_\mu(x)$ that leaves $\phi(x)$ inert. The quadratic in A term in the definition of $\hat{g}_{\mu\nu}$ guarantees that the 0-mode (z -independent part) of the D -dimensional metric $g_{\mu\nu}$ remains also invariant under $U(1)$ gauge transformations.

上述规范条件给出的结果是: 在度规分解 (2) 中，只有 D 维度规 $g_{\mu\nu}(x^m, z)$ 依赖所有 $(D+1)$ 坐标，而半径子 $\phi(x)$ 和引力光子 $A_\mu(x)$ 不依赖紧致坐标 z 。此外，剩余坐标变换仅依赖非紧致坐标 x^m : $\xi^\mu(x)$ ，它们生成 D 维坐标变换，而 $\xi^z(x)$ 对应引力光子 $A_\mu(x)$ 的 $U(1)$ 规范变换，且不改变 $\phi(x)$ 。$\hat{g}_{\mu\nu}$ 定义中的 A 二次项保证了 D 维度规 $g_{\mu\nu}$ 的零模 (不依赖 z 的部分) 在 $U(1)$ 规范变换下也保持不变。

We now focus on the simplest choice where the D -dimensional metric $g_{\mu\nu}$ and thus all fields are independent of the z coordinate. The action (1) gives then for the zero modes:

我们现在来看最简单的选择: D 维度规 $g_{\mu\nu}$ 以及所有场都不依赖 z 坐标。此时作用量 (1) 对零模给出:

$$S_{EH}^{(D+1)} = \frac{2\pi R}{2\hat{\kappa}^2} \int \mathrm{d}^D x \sqrt{(-1)^{D-1} g} e^{((D-2)a+b)\phi} \{ \mathcal{R} - [2(1-D)a - 2b] \Box \phi$$

$$- \left[(D-2)(1-D)a^2 + 2b((2-D)a - b) \right] (\partial\phi)^2$$

$$-\frac{1}{4}e^{2(b-a)\phi}F^2\Big\}\tag{8}$$

To have a canonical normalization of the D -dimensional Einstein-Hilbert action, we need

为了对 D 维爱因斯坦-希尔伯特作用量进行标准归一化，我们需要

$$(D-2)\,a+b=0.\tag{9}$$

while a canonical dilaton kinetic term requires:

而标准的 dilaton 动能项要求:

$$a^2=\frac{1}{2\,(D-1)\,(D-2)}.\tag{10}$$

In the following we choose the positive root. We can identify the D -dimensional Planck mass:

在下文中我们选取正根。我们可以得到 D 维普朗克质量的表达式:

$$\frac{1}{\kappa^2}=\frac{2\pi R}{\hat{\kappa}^2}\Rightarrow M_P^{D-2}=2\pi R\widehat{M}_P^{D-1}.$$

(11)

We are used to work with dimensionful fields instead of the dimensionless ϕ and A_μ . Therefore, we perform the rescaling:

我们通常使用带量纲的场而非无量纲的 ϕ 和 A_μ ，因此我们进行如下重标度:

$$\phi\to\sqrt{2}\kappa\phi;\ A_\mu\to\sqrt{2}\kappa A_\mu\tag{12}$$

leading to the action

得到作用量

$$\mathcal{S}^{(D)}=\int\mathrm{d}^Dx\sqrt{(-1)^{D-1}g}\left[\frac{\mathcal{R}}{2\kappa^2}+\frac{1}{\kappa\sqrt{(D-1)\,(D-2)}}\Box\phi+\frac{1}{2}(\partial\phi)^2-\frac{1}{4}e^{-2\sqrt{\frac{D-1}{D-2}}\kappa\phi}F^2\right]$$

(13)

where the second term is a total derivative that will not play any role for the purpose of our discussions.

其中第二项是全导数，在我们的讨论中不起作用

Let's now consider the corresponding dimensional reduction of a free real massless scalar field $\widehat{\Phi}$:

现在我们考虑自由实无质量标量场 $\widehat{\Phi}$ 对应的维数约化:

$$\mathcal{S}_{\Phi}^{(D+1)} = \int \mathrm{d}^{D+1}x\sqrt{(-1)^{D}\hat{g}}\frac{1}{2}\hat{g}^{MN}\partial_M\hat{\Phi}\partial_N\hat{\Phi} \tag{14}$$

Again, for simplicity, we consider the case where the field $\hat{\Phi}$ is periodic on the extra dimension:

为简化起见，我们再次考虑场 $\hat{\Phi}$ 在额外维上周期的情形:

$$\hat{\Phi}(x, z+2\pi R) = \hat{\Phi}(x,z)\,,\ \hat{\Phi}(x,z) = \frac{1}{\sqrt{2\pi R}}\sum_{n=-\infty}^{+\infty}\varphi_n(x)\,e^{\frac{inz}{R}}\,, \tag{15}$$

with $\varphi_{-n} = \varphi_n^*$, which leads to

其中 $\varphi_{-n} = \varphi_n^*$ ，由此可得

$$\begin{aligned}\mathcal{S}_{\Phi}^{(D)} = \int \mathrm{d}^{D}x\sqrt{(-1)^{D-1}g}\Bigg\{&\frac{1}{2}\partial_\mu\varphi_0\partial^\mu\varphi_0 + \sum_{n=1}^{\infty}\left(\partial_\mu\varphi_n\partial^\mu\varphi_n^* - \frac{n^2}{R^2}e^{2\sqrt{\frac{D-1}{D-2}}\kappa\phi}\varphi_n\varphi_n^*\right)\\ &+\sum_{n=1}^{\infty}\left(i\sqrt{2}\kappa\frac{n}{R}A^\mu\left(\partial_\mu\varphi_n\varphi_n^* - \varphi_n\partial_\mu\varphi_n^*\right) + 2\kappa^2\frac{n^2}{R^2}A_\mu A^\mu\varphi_n\varphi_n^*\right)\Bigg\},\end{aligned}$$

(16)

We can now comment our results for some generic lessons on extra dimensions (for a more detailed discussion, see, e.g., [11]):

现在我们可以对额外维的一些通用结论进行评述 (更详细的讨论参见例如文献 [11]):

- The complex scalars φ_n form a tower of massive states with masses:

- 复标量 φ_n 构成一列质量态塔，其质量为:

$$m_n = \frac{n}{R}e^{\sqrt{\frac{D-1}{D-2}}\kappa\phi}. \tag{17}$$

This is the Kaluza-Klein (KK) tower. In the very simple case of a circle, the masses are regularly spaced. This is not the case for any compactification. There are however two important generic properties that we would like to note. First the masses depend on the value in vacuum of a scalar field, here the dilaton ϕ . In the case of supersymmetric theories, these are moduli fields. Then, in the decompactification limit, by taking $R\to\infty$ or $\phi\to-\infty$, there is an infinity of states which become very light exponentially fast. This is one of the manifestations of the Swampland Distance Conjecture in string theory to which we will return below.

这就是卡鲁扎-克莱因 (KK) 态塔。在圆紧致化的简单情形中，质量是均匀间隔的，这一点对任意紧致化并不成立。但我们需要指出两个重要的通用性质: 首先，质量依赖于标量场的真空期望值，此处就是 dilaton ϕ 。在超对称理论中这些是模场。其次，在退紧致化极限下，当取 $R\to\infty$ 或 $\phi\to-\infty$ 时，有无穷多个态会指数级快速地变得极轻。这是弦理论中沼泽地距离猜想的表现之一，我们后文中会再讨论。

- The KK modes appear minimally coupled to the graviphoton. In general, internal space isometries give rise to gauge symmetries. In the particular case here, it is a $U(1)$ symmetry. The gauge coupling

- KK 模与引力光子是最小耦合。一般来说，内空间等度量对称性会诱导出规范对称性，在本文的特殊情形中，这是一个 $U(1)$ 对称性，规范耦合

$$g^2 = e^{2\sqrt{\frac{D-1}{D-2}}\kappa\phi}. \tag{18}$$

is again a function of a scalar field, here the dilaton ϕ , and taking $g \to 0$; thus $\phi \to -\infty$, we get an infinity of light states, again manifestations of the Swampland Distance Conjecture.

同样是标量场的函数，此处就是 dilaton ϕ ，当取 $g \to 0$ 从而得到 $\phi \to -\infty$ 时，我们会得到无穷多个轻态，这同样是沼泽地距离猜想的表现。

The charges of the KK states are given by

KK 态的电荷由下式给出

$$gq_n = \sqrt{2}\kappa\frac{n}{R}e^{\sqrt{\frac{D-1}{D-2}}\kappa\phi} \tag{19}$$

Mass and charge are related through

质量和电荷满足关系

$$(gq_n)^2 = 2\kappa^2 m_n^2, \tag{20}$$

saturating the BPS condition. This is expected as all the interactions unify to descend from the unique gravitational interaction of a free scalar field in higher dimensions.

满足 BPS 条件。这符合预期，因为所有相互作用统一后都源自高维自由标量场的独特引力相互作用。

- The same tower of KK modes appears also in the massive spin-2 excitations of the metric $g_{\mu\nu}(x^\rho, z)$ in the matrix decomposition (2). Note that in the unitary gauge choice where the radion $\phi(x)$ and the graviphoton $A_\mu(x)$ are independent of the internal compact coordinate z , only the metric $g_{\mu\nu}$ has KK excitations. The would-be $(D-2)+1$ excitations of A_μ and ϕ are absorbed by the KK modes of the metric to provide the longitudinal and scalar helicities of the massive spin-2. Indeed, the latter has $(D-1)D/2$ helicities, versus $(D-2)(D-1)/2$ of the massless case.

- 同样的 KK 模塔也出现在矩阵分解 (2) 中度规 $g_{\mu\nu}(x^\rho, z)$ 的有质量自旋-2 激发里。注意在 radion 场 $\phi(x)$ 引力光子 $A_\mu(x)$ 都不依赖于内部紧致坐标 z 的幺正规范选取下，只有度规 $g_{\mu\nu}$ 存在卡鲁扎-克莱因激发。原本属于 A_μ 和 ϕ 的 $(D-2)+1$ 激发会被度规的 KK 模吸收，为有质量自旋-2 提供纵螺旋度和标量螺旋度。实际上，后者共有 $(D-1)D/2$ 种螺旋度，而无质量情况只有 $(D-2)(D-1)/2$ 种。

- Finally, we have established in the equation a relation between the Planck masses in D and $D+1$ dimensions. This relation is generalized in the case of the factorization of any internal space with the Minkowski space-time. For a δ -dimensional internal space K with volume $\mathrm{Vol}(K)$, we have:

- 最后，我们在方程中建立了 D 维和 $D+1$ 维普朗克质量之间的关系。该关系可以推广到任意内部空间与闵氏时空直积的情况。对于体积为 $\mathrm{Vol}\,(K)$ 的 δ 维内部空间 K，我们有:

$$\frac{1}{\kappa^2} = \frac{\mathrm{Vol}\,(K)}{\hat{\kappa}^2} \Rightarrow M_P^{D-2} = \mathrm{Vol}\,(K)\,\widehat{M}_P^{D+\delta-2} \tag{21}$$

which for the case where K is a torus of equal radii R reads

当 K 是半径相等的环面 R 时，上式可写为

$$\frac{1}{\kappa^2} = \frac{(2\pi R)^\delta}{\hat{\kappa}^2} \Rightarrow M_P^{D-2} = (2\pi R)^\delta\,\widehat{M}_P^{D+\delta-2} \tag{22}$$

This implies in particular that the large value of the D dimensional Planck mass (compared with a given energy scale, e.g., the electroweak scale) might be a consequence of large volume, while the $D+\delta$ dimensional Planck mass is much smaller.

这尤其说明: D 维普朗克质量 (相较特定能标，例如电弱能标) 数值很大，可能是大体积带来的结果，而 $D+\delta$ 维普朗克质量要小得多。

This last observation has important consequences. For example, as the four-dimensional Planck scale, M_P is related to the ”fundamental Planck scale” $\widehat{M}_P$ through

最后这个结论有重要意义。例如，四维普朗克标度 M_P 与“基本普朗克标度”$\widehat{M}_P$ 通过下式关联

$$M_P^2 \sim R^\delta \widehat{M}_P^{2+\delta} \tag{23}$$

the existence of large extra dimensions with size $R\widehat{M}_P \gg 1$ allows for a low fundamental scale. The fundamental scale can be as low as the TeV [9, 10] , or at intermediate energies [12, 13, 53] .

尺寸为 $R\widehat{M}_P \gg 1$ 的大额外维度的存在允许基本标度很低。基本标度可以低至 TeV 量级 [9, 10]，也可以处于中间能区 [12, 13, 53] 。

Before mentionning experimental investigations for extra dimensions, we still need to discuss two important aspects of compactifications.

在讨论额外维度的实验研究之前，我们仍需要讨论紧致化的两个重要方面。

One aspect is called T -duality. When the size of a dimension tends to zero, the KK modes become very massive. However, in string theories, there are states whose mass decreases when the size of additional dimensions decreases. These are the winding states, whose masses are given by

其中一个方面被称为 T 对偶。当额外维度的尺寸趋近于零时，KK 模会变得极重。但在弦论中，存在一类随额外维度尺寸减小质量反而降低的态。这类态就是缠绕态，其质量由下式给出

$$\widetilde{m}_l = lRe^{-\sqrt{\frac{D-1}{D-2}}\kappa\phi}, \tag{24}$$

where the integer l is the winding number of the closed string around the compactification circle. The simplest form of T -duality corresponds to the inversion of the compactification radius (in string units) together with the exchange of the KK and winding modes.It turns out that this is a symmetry of the closed string spectrum and interactions.

其中整数 l 是闭弦缠绕紧致化圆周的缠绕数。T 对偶最简单的形式对应于 (弦单位下) 紧致化半径取逆，同时交换 KK 模与缠绕模。这恰好是闭弦谱和相互作用的一个对称性。

The other aspect concerns the existence of points in the inner space where certain states are localized. These can be particular points in this space, but can also be edges. From the point of view of point particle theory, these points seem to correspond to singularities, but these are resolved in string theory. Examples are either fixed points of an orbifold, or branes. The simplest example of such a case is an internal space described as a segment. In this case, some states would be localized at its edges. An important property then is that since these points break the translation invariance in the corresponding directions of the internal space, the momentum along them is not conserved. Therefore, single KK modes can decay to, or be produced from, states localized there. For completeness, we describe below the main properties of orbifold compactifications, restricting to the simplest case of a line interval. Orbifolds The main motivation comes from generating chirality, since toroidal compactifications give rise to non-chiral theories since a five-dimensional spinor is reduced to a Dirac fermion in four dimensions. Emergence of chirality requires internal manifolds with nontrivial holonomy, while orbifolds are special (singular) limits obtained by quotient of a regular manifold by a discrete symmetry. The resulting spaces have conical singularities at the fixed points of the discrete symmetry transformations. The simplest one-dimensional example is the line interval I , which can be obtained by quotient a circle S^1 by the Z_2 parity $z \to -z, I = S^1/Z_2$. The two end-points of the interval, $z = 0$ and $z = \pi R$, are fixed points under Z_2 .

另一方面则关乎内部空间中存在特定定域态的位置。这些位置可以是内部空间中的特定点，也可以是边界。从点粒子理论的角度看，这些位置似乎对应奇点，但该问题在弦论中得到了解决。这类位置的例子包括轨形的不动点, 或是膜。最简单的例子是被描述为线段的内部空间, 在这种情况下, 部分态会定域在线段的边界。一个重要性质是: 由于这些位置破坏了内部空间对应方向上的平移不变性，沿这些方向的动量不守恒，因此单个卡鲁扎-克莱因 (KK) 模可以衰变生成定域于此的态，也可由这些态产生。为求完整，我们在下文中介绍轨形紧致化的主要性质，并限定讨论线间隔这一最简单的情况。## 轨形轨形紧致化的核心动机是得到手征性: 因为环面紧致化只会生成非手征理论——五维旋量约化到四维后会得到一个狄拉克费米子。手征性的出现要求内部流形具有非平凡和乐，而轨形是对正则流形做离散对称性商得到的特殊 (奇异) 极限。得到的空间在离散对称变换的不动点处存在圆锥奇点。最简单的一维例子是线间隔 I ，它可以通过对圆 S^1 做 Z_2 宇称 $z \to -z, I = S^1/Z_2$ 商得到。该线间隔的两个端点 $z = 0$ 和 $z = \pi R$ 是 Z_2 下的不动点。

Since Z_2 is a symmetry of the higher-dimensional theory, five dimensional fields should be even or odd under the Z_2 parity. They can thus be expanded in terms of $\cos nz/R$ or $\sin nz/R$, instead of plane waves in (15):

由于 Z_2 是高维理论的对称性，五维场在 Z_2 宇称变换下应为偶宇称或奇宇称。因此它们可以按 $\cos nz/R$ 或 $\sin nz/R$ 展开，而非式 (15) 中的平面波展开:

$$\hat{\Phi}_{\text{even}}(x,-z)=\hat{\Phi}_{\text{even}}(x,z):\hat{\Phi}_{\text{even}}(x,z)=\frac{1}{\sqrt{\pi R}}\varphi_0^e(x)+\sqrt{\frac{2}{\pi R}}\sum_{n=1}^{+\infty}\varphi_n^e(x)\cos\frac{nz}{R}$$

$$\hat{\Phi}_{\text{odd}}(x,-z)=-\hat{\Phi}_{\text{odd}}(x,z):\hat{\Phi}_{\text{odd}}(x,z)=\sqrt{\frac{2}{\pi R}}\sum_{n=1}^{+\infty}\varphi_n^o(x)\sin\frac{nz}{R}, \tag{25}$$

where the normalization is fixed for $z\in[0,\pi R]$. Note that only the even fields have four-dimensional zero modes. Actually, there is an alternative way of studying orbifolds by imposing boundary conditions at the ”end of the world” boundaries, which in our example are the two end-points of the interval. The even fields correspond to Neumann boundary conditions where the derivatives of the fields vanish at the end-points, while the odd fields correspond to Dirichlet boundary conditions where the fields vanish at the end-points:

其中归一化由 $z\in[0,\pi R]$ 确定。注意只有偶场存在四维零模。实际上，还可通过在“世界尽头”边界施加边界条件来研究轨形，在我们的例子中该边界就是区间的两个端点。偶场对应诺依曼边界条件，即场的导数在端点处为零；而奇场对应狄利克雷边界条件，即场本身在端点处为零:

$$\partial_z\hat{\Phi}_{\text{even}}(x,z=0)=\partial_z\hat{\Phi}_{\text{even}}(x,z=\pi R)=0 \tag{26}$$

$$\hat{\Phi}_{\text{odd}}(x,z=0)=\hat{\Phi}_{\text{odd}}(x,z=\pi R)=0 \tag{27}$$

Consider next a five-dimensional gauge field $\hat{A}_M(x,z)$ associated with a gauge transformation:

接下来考虑与规范变换相关联的五维规范场 $\hat{A}_M(x,z)$:

$$\delta_\omega\hat{A}_M(x,z)=\partial_M\omega(x,z). \tag{28}$$

Choosing the gauge parameter ω to be even under $z\to -z$, it follows that $\hat{A}_\mu$ is even while $\hat{A}_z$ is odd. Moreover, we can gauge-fix the ω transformation for $\partial_z\omega\neq 0$ by taking $\partial_z A_z=0$, implying that A_z vanishes since it is odd. As a result, one is leftover with $\hat{A}_\mu$ which has a KK mode expansion as an even field in (25) with its four-dimensional vector 0-mode $A_\mu(x)^{(0)}$ associated with a four-dimensional gauge transformation $\omega(x)$.

选取规范参数 ω 在 $z\to -z$ 变换下为偶，由此可得 $\hat{A}_\mu$ 为偶，而 $\hat{A}_z$ 为奇。此外，我们可以通过取 $\partial_z A_z=0$ 对 $\partial_z\omega\neq 0$ 的 ω 变换进行规范固定，这意味着 A_z 因是奇场而消失。最终剩余场为 $\hat{A}_\mu$ ，作为偶场它具备式 (25) 所示的卡鲁扎-克莱因模展开，其四维矢量零模 $A_\mu(x)^{(0)}$ 对应一个四维规范变换 $\omega(x)$ 。

Let us consider now a five-dimensional massless fermion $\hat{\Psi}(x,z)$; its transformation under Z_2 has the form:

现在我们来考虑一个五维无质量费米子 $\hat{\Psi}(x,z)$ ；它在 Z_2 下的变换形式为:

$$\hat{\Psi}(x,-z)=\Gamma\hat{\Psi}(x,z)\ ;\ \Gamma^2=1. \tag{29}$$

Since the vector current $\overline{\hat{\Psi}}\gamma^M\hat{\Psi}$ must transform as $\hat{A}_M$, $\overline{\hat{\Psi}}\gamma^\mu\hat{\Psi}$ must be event while $\overline{\hat{\Psi}}\gamma^5\hat{\Psi}$ must be odd. It follows that $\Gamma = \gamma^5$, implying that the four-dimensional zero mode of $\hat{\Psi}$ is chiral. Thus, orbifold compactifications lead to chirality.

由于矢量流 $\overline{\hat{\Psi}}\gamma^M\hat{\Psi}$ 必须按变换规则变换，$\hat{A}_M$, $\overline{\hat{\Psi}}\gamma^\mu\hat{\Psi}$ 必须为偶宇称，而 $\overline{\hat{\Psi}}\gamma^5\hat{\Psi}$ 必须为奇宇称。由此可得 $\Gamma = \gamma^5$ ，这意味着 $\hat{\Psi}$ 的四维零模是手征的。因此，轨形紧致化可以产生手征性。

Searches for Extra Dimensions

额外维度搜索

In the previous section, we have pointed out manifestations of extra dimensions: the KK modes and modification of gravitational interaction by the presence of a lower fundamental Planck mass. We shall use these two facts here to search for experimental signatures of extra dimensions.

在前一节中，我们已经指出了额外维度的表现形式: 卡鲁扎-克莱因模态 (KK 模态)，以及因更低的基本普朗克质量存在而产生的引力相互作用修正。我们将在这里利用这两个性质，寻找额外维度的实验信号。

Tabletop Experiments

桌面实验

From now on, we consider a space-time of dimension $4 + d$ which factorizes as $3 + 1$ non-compact dimensions times an internal space K of dimension d .

接下来我们讨论维度为 $4 + d$ 的时空，它可分解为 $3 + 1$ 个非紧致维度乘以维度为 d 的内空间 K 。

At large length scales, the typical energies of particles are far too small to produce on-shell Kaluza-Klein modes. No signal propagates in K ; the space-time appears four-dimensional. By increasing the energy, we eventually reach values sufficient to create KK excitations, and we then start to probe the K space. At length scales much smaller than the radii of δ compact dimensions, among the d of K , the gravitational interactions are locally well approximated by a $4 + \delta$ dimensional description. Applying the Gauss law, one expects a transition from a gravitational potential from an $1/r$ behavior to an $1/r^{1+\delta}$. At lengths comparable to the compactification scale, the gravitational potential can be parametrized as

在大尺度下，粒子的典型能量远低于产生在壳卡鲁扎-克莱因 (Kaluza-Klein，KK) 模式所需的能量，没有信号能在 K 维时空中传播，因此时空表现为四维。当能量升高，我们最终会获得足以产生 KK 激发态的能量，进而开始探测 K 维空间。在远小于 δ 个紧致维度半径的尺度下，在 K 的内空间 d 中，引力相互作用在局域上可以很好地用 $4 + \delta$ 维描述近似。根据高斯定律，我们预期引力势会从 $1/r$ 行为转变为 $1/r^{1+\delta}$ 行为。在和紧致化尺度相当的长度下，引力势可以参数化为

$$V(r) = -G_N \frac{m_1 m_2}{r} \left(1 + \alpha e^{-r/\lambda}\right), \tag{30}$$

where $\lambda = 1/m$ is the wavelength of the lighter KK mode of mass m . For δ extra dimensions, one has:

其中 $\lambda = 1/m$ 是质量为 m 的最轻 KK 模式的波长，对于 δ 个额外维度，有:

$$\alpha = \frac{8\delta}{3}, \tag{31}$$

providing the strength of the extra Yukawa-type force relative to gravity.

它给出了额外汤川型力相对于引力的强度。

Many tabletop experiments have been designed to investigate the existence of new forces at very short distances (for a review see, e.g., [14]). In particular, the tightest limits for extra forces of strength comparable to gravity ($\alpha \sim \mathcal{O}(1)$) are obtained by torsional pendulum kind of experiments. Basically, in this type of experiments, the idea is that a torsion pendulum is holding a disc placed over another rotating attractor disc. The upper disc is put in movement by twisting the cable of the pendulum, while the lower one is rotating at constant speed. Due to the presence of holes on the discs, there is a contribution to the torque when all the holes are not facing each other. The hanging disc feels a periodic torque that can be measured with a very high precision. In fact, one measures a set of oscillation frequencies and compare with expectation. The measured torques are now consistent with a purely inverse square law interaction and thus allow us to put bounds on new forces and possible large extra dimensions.

人们设计了大量桌面实验来极短距离下研究新力的存在 (综述可见例如文献 [14])。其中，强度和引力相当的额外力 ($\alpha \sim \mathcal{O}(1)$) 的最严格限制来自扭摆类实验。这类实验的基本思路是: 扭悬摆托着一个圆盘，圆盘下方是另一个旋转的吸引盘，扭转摆线带动上方圆盘运动，下方圆盘匀速旋转。由于两个圆盘都开有孔，当孔不完全对齐时，就会对扭矩产生贡献。悬挂的圆盘会感受到周期性扭矩，该扭矩可以极高精度测量。实验中会测量一组振荡频率并和理论预期对比。目前测量得到的扭矩与纯平方反比律相互作用一致，因此可以对新力和可能的大额外维度给出限制。

The forces we are trying to measure accurately are gravitational in nature and therefore very weak. There are many sources of background and noise to contend with, which one needs to be able to evaluate theoretically and reduce their importance in the experiments. We do not discuss them in detail here. Instead, we just very briefly mention how the treatment of certain background sources limits the domain that can be probed by these experiments. Firstly, in order to avoid the presence of Coulomb forces that cannot be evaluated, conductive materials are used. Secondly, to minimize the effects of potential differences between the crystals at different locations on the facing surfaces, a gold coating is used. Finally, to reduce all these effects as well as the effects of Casimir forces, the detector and attractor surfaces are moving relative to each other, but also a stiff, stationary, conductive membrane is placed between them. To improve the minimum separation between the pendulum and the attractor has been a challenge because of sensitivity to alignment uncertainties, vibrations, and even dust particles. The thickness of the membrane means however that it is not possible to go to distances of less than about ten microns. All together with an important improvement in modeling and data analysis during the years has allowed to get to an impressive sensitivity. The strongest limit for $\delta = 2$ extra dimensions is today of order $R \lesssim 30\mu$m [15, 16] .

我们试图精确测量的力本质上是引力，因此非常微弱。实验中存在大量背景和噪声源，需要从理论上评估并降低它们的影响，本文不对这些内容展开讨论，仅简要介绍对背景源的处理如何限制了这类实验可探测的范围。首先，为了避免无法计算的库仑力，实验使用导电材料。其次，为了减小相对表面不同位置晶体间电势差的影响，会使用金镀层。最后，为了减小上述效应以及卡西米尔力的影响，探测器和吸引盘表面不仅会相对运动，还会在二者之间放置一块刚性固定的导电薄膜。由于对对准误差、振动甚至灰尘都很敏感，缩小扭摆和吸引盘之间的最小间隔一直是一项挑战。而薄膜的厚度决定了间隔无法缩小到约十微米以下。经过多年来模型和数据分析的持续改进，实验已经获得了极高的灵敏度。目前对 $\delta = 2$ 额外维度的最强限制约为 $R \lesssim 30\mu$m [15, 16] 。

Astrophysics and Cosmology Bounds

天体物理与宇宙学限制

Models with extra dimensions are subject to cosmological and astrophysical constraints [17-19].

额外维模型受到宇宙学与天体物理的约束 [17-19]。

The first constraint is that the gravitons in the bulk should not store too much energy. Relic gravitons should not close the Universe: this constrains, for example, the fundamental scale to $\gtrsim$ 7TeV for $\delta = 2$ extra dimensions.

第一个约束是，体空间中的引力子不能储存过多能量。遗迹引力子不能导致宇宙闭合: 例如，这就将额外维 $\delta = 2$ 对应的基础标度限制为 $\gtrsim$ 7TeV 。

Then, these KK relic gravitons can decay into photons and contribute to the cosmic diffuse gamma ray. By imposing that the models do not predict a too bright sky, one obtains limits on the fundamental scale, e.g., for $\delta = 2$ extra dimensions, the fundamental scale of the theory must be $\gtrsim$ 100TeV [18, 19] .

此外，这些 KK 遗迹引力子可以衰变为光子，对宇宙弥散伽马射线有贡献。通过要求模型不会预言过亮的天空，可得到基础标度的限制: 例如，当额外维为 $\delta = 2$ 时，理论的基础标度必须为 $\gtrsim$ 100TeV [18, 19] 。

Fairly light KK excitations of gravitons are abundantly produced in the interior of stars. These particles carry a quantity of energy which can be non-negligible. The agreement of stellar models with observations leads to limits on the number of such KK states. The limit on the fundamental scale from the supernova SN1987A is of order of 27TeV for $\delta = 2$ extra dimensions [20]. After a supernova explosion, the least energetic KK gravitons remain gravitationally trapped in the remaining neutron star. The requirement that neutron stars should not be excessively heated by the decays of these KK modes into photons gives the strongest limit such as 1740 TeV for $\delta = 2$ extra dimensions [21], excluding this scenario as a solution for the electroweak versus Planck scale hierarchy problem.

质量相当小的引力子 KK 激发会在恒星内部大量产生，这些粒子携带的能量不可忽略。恒星模型与观测的一致性对这类 KK 态的数量给出了限制。来自超新星 SN1987A 的基础标度限制，对于 $\delta = 2$ 个额外维约为 27TeV [20]。超新星爆发后，能量最低的 KK 引力子会被残余中子星引力束缚，而这些 KK 模式衰变为光子不会过度加热中子星，这一要求给出了最强的限制，例如对于 $\delta = 2$ 个额外维，限制为 1740 TeV[21]，排除了该模型作为电弱标度与普朗克标度等级问题解决方案的可能。

Searches at Collider Experiments

对撞机实验中的搜寻

Analyses of the data collected from experiments at colliders allow to put other bounds on the size of extra dimensions (A detailed discussion can be found in the Particle Data Group section on extra dimensions [27].). The main searches can be separated into two categories: searches for Standard Model particles, in particular gauge bosons [8,22], their KK excitations on one side, and graviton KK modes [23- 26] on the other side. In each case, one can search either for on-shell production or for virtual exchange effects. With increasing energy, the string substructure can also be excited, leading to stringy effects at colliders. These can be on-shell production, or give rise to dimension 6 or 8 effective operators.

对从对撞机实验收集的数据的分析可以对额外维度的大小给出其他限制 (详细讨论可在粒子数据小组关于额外维度的章节中找到 [27]。) 主要的搜寻可分为两类: 一类是搜寻标准模型粒子，尤其是规范玻色子 [8,22] 的卡鲁扎-克莱因 (KK) 激发，另一类是搜寻引力子 KK 模式 [23-26]。两类情况中，都既可以搜寻 on-shell 产生，也可以搜寻虚交换效应。随着能量升高，弦亚结构也可被激发，在对撞机上产生弦效应。这些效应可以是 on-shell 产生，也可以引发 6 维或 8 维有效算符。

The graviton KK excitations produced in colliders interact very weakly and thus appear as missing transverse energy. Following the non-observation of this missing energy in jet production, ATLAS [28] and CMS [29] give limits of the same order, with 139 (137) fb^{-1} :

对撞机中产生的引力子 KK 激发相互作用非常弱，因此会表现为缺失横能。在喷注产生过程中未观测到这种缺失能量后，ATLAS[28] 和 CMS[29] 给出了同量级的限制，在 139(137) fb^{-1} :

$$M_{P,4+\delta} \gtrsim 5.5 - 11\mathrm{TeV}\ \delta = 6 - 2. \tag{32}$$

ATLAS [30] and CMS [31] bounds on the fundamental scale, from nonobservation of string resonance modes in di-jets, are

ATLAS[30] 和 CMS[31] 通过未在双喷注中观测到弦共振模式，给出的基本标度限制为

$$M_{\text{string}} \gtrsim 8\mathrm{TeV} \tag{33}$$

A priori, one might think that a limit on the fundamental scale can be derived by considering that the exchange of virtual modes of KK gravitons will induce an effective operator of the form:

乍看之下，有人可能会认为可以通过如下思路推导基本标度的限制:KK 引力子的虚模式交换会诱发形式如下的有效算符:

$$\frac{4}{\Lambda}\left(T_{\mu\nu}T^{\mu\nu}-\frac{1}{\delta+2}T_{\mu}^{\mu}T_{\nu}^{\nu}\right) \tag{34}$$

where $T_{\mu\nu}$ is the stress-energy tensor and Λ is of the order of the fundamental Planck mass. CMS (2018) bounds the size of this operator from analysis of the dijet angular distribution with 35.9fb^{-1} to be of order [32]:

其中 $T_{\mu\nu}$ 是能量动量张量，Λ 的量级为基本普朗克质量。CMS(2018 年) 通过分析 35.9fb^{-1} 下的双喷注角分布，将该算符的大小限制在量级 [32]:

$$\Lambda \gtrsim 9.1\text{TeV} \tag{35}$$

However, this limit should not be taken too seriously. In any quantum theory of gravity, one expects to see new states, new interactions at this scale. These will induce new effective operators of dimension 8, or worse of dimension 6, which will pollute or dominate the virtual graviton exchange operators. This has been studied in particular for string theory in [33-35]. These operators are therefore too polluted to extract robust bounds.

但这个限制并不严谨。在任何量子引力理论中，我们都预期会在该标度看到新态、新相互作用。这些会诱发 8 维，甚至 6 维的新有效算符，干扰或主导虚引力子交换算符。弦理论中已在文献 [33-35] 中对此做了专门研究。因此原算符受干扰过大，无法得到可靠的限制。

Another prediction of extra-dimensional models is the existence of KK excited states of Standard Model particles. We will not discuss here in detail the different possibilities, but we will comment on the case of KK gauge bosons.

额外维度模型的另一个预言是存在标准模型粒子的 KK 激发态。本文不详细讨论不同可能性，仅对 KK 规范玻色子的情况做说明。

The simplest case corresponds to the KK modes which appear as a sequential tower of Z,' W', gluons,' and photons' which interact with localized states at the boundary. Limits were derived from the analysis of the 2018 LHC data (36 fb-1). Rescaling the ATLAS [36] and CMS [37] experiment bounds would give a limit roughly in the range $\frac{1}{R} \gtrsim 10\text{TeV}$.

最简单的情况对应 KK 模式表现为顺序出现的 Z'、W'、胶子' 和光子' 塔, 它们与边界上的定域态相互作用。人们已经通过分析 2018 年 LHC 数据 ($36\ \text{fb}^{-1}$) 得到了限制。重新标度 ATLAS[36] 和 CMS[37] 的实验限制后，得到的大致限制范围为 $\frac{1}{R} \gtrsim 10\text{TeV}$ 。

How Many KK Modes?

有多少个卡鲁扎-克莱因模?

In the case of on-shell production of KK modes, their number is determined by the available energy and phase space. But, in the case of an exchange of virtual KK modes, one has an infinite number that can be exchanged. At first sight, the contribution of the sum then seems to diverge [7]. Indeed, let us consider the sum over the propagators, for example, of modes exchanged in the $s \to 0$ channel of a $2 \to 2$ interaction. This reads:

在卡鲁扎-克莱因模的在壳产生情形下，其数量由可用能量和相空间决定。但在虚卡鲁扎-克莱因模交换的情形下，可交换的卡鲁扎-克莱因模数量是无穷多的。乍看之下，这个求和的贡献似乎会发散 [7]。我们来实际考虑一下传播子的求和，例如 $s \to 0$ 相互作用 $2 \to 2$ 道中交换模式的传播子求和，形式如下:

$$\sum_{n_1,\cdots,n_d} \frac{g_{\mathbf{n}}^2}{-s + m_0^2 + n_1^2 + \cdots + n_d^2} \tag{36}$$

This sum diverges for $d > 1$ if the couplings $g_{\mathbf{n}}$ of the KK modes are independent of $\mathbf{n}$. Often in the literature, one finds that the sum is made finite by cutting off the number of KK states, to take into account only those below the fundamental Planck mass. However, as we will see in the next section, the whole infinite tower of KK modes is required in order to preserve all symmetries of the theory, and such a truncation is not justified. We then turn to string theory where such sums exist and give finite and consistent results. Let us suppose for instance that we exchange the KK excitations of gauge bosons [7]:

若卡鲁扎-克莱因模的耦合 $g_{\mathbf{n}}$ 与 $\mathbf{n}$ 无关，该求和对 $d > 1$ 发散。文献中常见的处理是对卡鲁扎-克莱因态的数量做截断，只保留基本普朗克质量以下的态，让求和变为有限。但我们会在下一节看到，为了保留理论的所有对称性，必须保留整个无穷的卡鲁扎-克莱因模塔，这种截断并不合理。接下来我们转向弦论，这类求和在弦论中存在并给出有限且自洽的结果。例如我们假设交换的是规范玻色子的卡鲁扎-克莱因激发 [7]:

$$A^{\mu}(x, \mathbf{y}) = \sum_{\mathbf{n}} \mathcal{A}_{\mathbf{n}}^{\mu}(x) \exp\left\{i \frac{n_i y_i}{R_i}\right\} \tag{37}$$

The result can be described by an effective theory where the coupling of the gauge bosons to the localized charge current takes the form [35]:

该结果可以用有效理论描述，其中规范玻色子与定域电荷流的耦合形式如下 [35]:

$$\int d^4x \sum_{\mathbf{n}} e^{-\ln\Delta \sum_i \frac{n_i^2 l_s^2}{2R_i^2}} j_{\mu}(x) \mathcal{A}_{\mathbf{n}}^{\mu}(x), \tag{38}$$

After Fourier transform, this reads:

傅里叶变换后，形式为:

$$\int d^4y \int d^4x \left(\frac{1}{l_s^2 2\pi \ln\Delta}\right)^2 e^{-\frac{\mathbf{y}^2}{2 l_s^2 \ln\Delta}} j_{\mu}(x) A^{\mu}(x, \mathbf{y}) \tag{39}$$

The localized matter is then felt as a Gaussian distribution of charge:

定域物质随后会表现为高斯电荷分布:

$$e^{-\frac{y^2}{2\sigma^2}} j_\mu(x) \ \sigma = \sqrt{\ln \Delta} l_s \sim 1.66 l_s \tag{40}$$

where $\Delta > 1$ is a model/compactification-dependent parameter defining the localization width. A similar parameter enters in the coupling of KK gravitons to D-branes.

其中 $\Delta > 1$ 是依赖于模型/紧致化的参数，定义了定域宽度。卡鲁扎-克莱因引力子与 D 膜的耦合中也会出现类似参数。

Symmetry Breaking Through Extra Dimensions

通过额外维度实现对称性破缺

Suppose that to hide the extra dimensions, one ”chooses” (We have seen above that the size of the extra dimensions is a vacuum expectation value of the redion field, which must be dynamically determined by the minimization of a corresponding scalar potential.) a very small size for the compactification radii. The natural scale would of course be of the order of the string scale $\frac{1}{R} \sim M_s$, which is then roughly $\frac{1}{R} \sim M_P$. The isometries of the compactification space allow us to obtain gauge symmetries, for example, in section ”Kaluza-Klein Excitations” we obtained a $U(1)$ gauge symmetry. Unfortunately, as noted by Oskar Klein, a quantized elementary charge implies particles of mass $\sim \frac{1}{R} \sim M_P$. Note, however, that this is a consequence of having imposed periodicity of the fields along the internal directions, an unnecessary condition as we will see for two cases in this section.

假设为了隐藏额外维度，人们「选择」(我们此前已经说明，额外维度的尺寸是红移场的真空期望值，必须通过最小化对应标量势动态确定) 让紧致化半径取非常小的值。自然的尺度当然是弦尺度 $\frac{1}{R} \sim M_s$ 量级，弦尺度大致为 $\frac{1}{R} \sim M_P$ 。紧致化空间的等距对称性允许我们得到规范对称性，例如在「卡鲁扎-克莱因激发」一节中我们得到了 $U(1)$ 规范对称性。遗憾的是，正如奥斯卡·克莱因指出的，量子化的元电荷要求粒子质量为 $\sim \frac{1}{R} \sim M_P$ 。但请注意，这是我们对场沿内空间方向施加周期性条件得到的结果，我们会在本节的两个案例中说明，这个条件并非必要。

Wilson Lines and Gauge Symmetry Breaking

威尔逊线与规范对称性破缺

For a scalar field Φ , the four-dimensional effective potential can be written in the Schwinger representation as

对于标量场 Φ ，四维有效势可以在施温格表示下写为

$$V_{eff}(\Phi) = \frac{1}{2} \sum_I (-)^{F_I} \int \frac{d^4 p}{(2\pi)^4} \log \left[p^2 + M_I^2(\Phi) \right]$$

$$= -\frac{1}{2}\sum_I (-)^{F_I}\int_0^\infty \frac{dt}{t}\int \frac{d^4p}{(2\pi)^4} e^{-t\left[p^2+M_I^2(\Phi)\right]}$$

$$= -\frac{1}{32\pi^2}\sum_I (-)^{F_I}\int_0^\infty \frac{dt}{t^3} e^{-tM_I^2(\Phi)}$$

$$= -\frac{1}{32\pi^2}\sum_I (-)^{F_I}\int_0^\infty dl l e^{-M_I^2(\Phi)/l} \tag{41}$$

where the sum is over all bosonic ($F_I = 0$) and fermionic ($F_I = 1$) degrees of freedom with Φ -dependent masses $M_I(\Phi)$, and we have made the change of variables $t = 1/l$. It is useful to keep in mind that the ultraviolet (UV) and infrared (IR) contributions correspond to the integration regions $t \to 0 (l \to \infty)$ and $t \to \infty \ (l \to 0)$, respectively.

其中求和遍历所有依赖 Φ 获得质量 $M_I(\Phi)$ 的玻色子 ($F_I = 0$) 和费米子 ($F_I = 1$) 自由度，我们已经做了变量替换 $t = 1/l$ 。需要牢记的是: 紫外线 (UV) 贡献和红外线 (IR) 贡献分别对应积分区域 $t \to 0 (l \to \infty)$ 和 $t \to \infty \ (l \to 0)$ 。

Here we are interested in a peculiar case of scalar fields: Wilson lines that descend from the dimensional reduction of a vector $\widehat{A}_M$ in higher dimensions (The material presented here follows the work [38,39].). To illustrate this, let us consider the action:

我们这里关注一类特殊的标量场: 源自高维矢量场 $\widehat{A}_M$ 维数约化的威尔逊线 (本文内容基于文献 [38,39] 整理)。为说明这一点，我们考虑如下作用量:

$$\mathcal{S} = \int \mathrm{d}^5x\sqrt{(-1)^D\hat{g}}\left\{\widehat{D}_M\widehat{\Phi}\widehat{D}^M\widehat{\Phi}^* - \widehat{M}^2\widehat{\Phi}\widehat{\Phi}^* - \frac{1}{4}\widehat{F}_{MN}\widehat{F}^{MN}\right\}, \tag{42}$$

where $\widehat{F}_{MN}$ is the field strength of $\widehat{A}_M$. We consider that $\widehat{A}_M$ is periodic:

其中 $\widehat{F}_{MN}$ 是 $\widehat{A}_M$ 的场强。我们假设 $\widehat{A}_M$ 是周期性的:

$$\widehat{A}_M(x, z + 2\pi R) = \widehat{A}_M(x, z) \Rightarrow \widehat{A}_M(x, z) = \frac{1}{\sqrt{2\pi R}}\sum_{n=-\infty}^{+\infty} A_{(n)M}(x)\, e^{\frac{inz}{R}} \tag{43}$$

while Φ changes by a phase:

而 Φ 会带有一个相位变化:

$$\widehat{\Phi}(x, z + 2\pi R) = e^{2i\pi q_\Phi \omega}\widehat{\Phi}(x, z) \Rightarrow \widehat{\Phi}(x, z) = \frac{1}{\sqrt{2\pi R}}\sum_{n=-\infty}^{+\infty} \varphi_n(x)\, e^{i(n+q_\Phi\omega)\frac{z}{R}},$$

(44)

where q_Φ is the $U(1)$ charge of Φ and ω an arbitrary constant. This boundary condition is allowed since the five-dimensional theory has a global $U(1)$ symmetry (as part of the gauge symmetry). Let us now consider a nonvanishing internal component of the gauge field $< A_{(0)5}(x) >= h_0 \neq 0$. The mass term in the Lagrangian for the scalar KK modes takes then the form:

其中 q_Φ 是 Φ 的 $U(1)$ 荷，ω 是任意常数。该边界条件是允许的，因为五维理论具有整体 $U(1)$ 对称性 (它是规范对称性的一部分)。现在我们考虑非零的规范场内部分量 $< A_{(0)5}(x) >= h_0 \neq 0$ 。卡鲁扎-克莱因 (KK) 标量模式的拉格朗日量中的质量项可写为如下形式:

$$\left(\hat{M}^2 + \left[\frac{n+a}{R}\right]^2\right)|\varphi_n|^2 \tag{45}$$

where a is the Wilson line given by

其中 a 是威尔逊线，定义为

$$a = q_\Phi \omega - q_\Phi g h_0 R = q_\Phi \left(\omega - g \oint \frac{dy^i}{2\pi} A_i\right). \tag{46}$$

It is straightforward to generalize to the case of d extra dimensions compactified on circles with radii $R_i > 1$, in units of the fundamental (string) length scale, with $i = 1, \ldots, d$. Then, equation (45) becomes:

可以很直接地推广到 d 个额外维紧致化在半径为 $R_i > 1$ 的圆上的情形，以基本 (弦) 长度标度为单位，满足 $i = 1, \ldots, d$ 。此时，方程 (45) 变为:

$$M^2_{\mathbf{m},I} = M^2_I(\phi) + \sum_{i=1}^{d} \left[\frac{m_i + a^I_i(\phi)}{R_i}\right]^2 \tag{47}$$

where I labels bosonic, but also fermionic, fields and $\mathbf{m} = \{m_1, \cdots, m_d\}$ with m_i integers, while the y^i coordinates parametrize the d -dimensional torus.

其中 I 标记玻色场与费米场，$\mathbf{m} = \{m_1, \cdots, m_d\}$ 满足 m_i 为整数，y^i 坐标参数化 d 维环面。

In the following, we take $M^2_I = 0$, leading to the one-loop effective potential:

在下文中，我们取 $M^2_I = 0$ ，得到单圈有效势:

$$V_{\text{eff}}(\phi)|_{\text{torus}} = -\sum_I \sum_{\mathbf{m}} (-)^{F_I} \frac{1}{32\pi^2} \int_0^\infty dl l e^{-\sum_i \frac{(m_i + a^I_i)^2}{R_i^2 l}} . \tag{48}$$

We start by commuting the integral with the sum over the KK states. As the number of KK states is infinite, we can perform a Poisson resummation:

我们首先交换对 KK 态的积分与求和顺序。由于 KK 态数量无限，我们可以做泊松求和:

$$V_{eff}(\phi)|_{\text{torus}} = -\sum_I (-)^{F_I} \frac{\prod_{i=1}^{d} R_i}{32\pi^{\frac{4-d}{2}}} \sum_{\mathbf{n}} e^{2\pi i \sum_i n_i a^I_i} \int_0^\infty dl l^{\frac{2+d}{2}} e^{-\pi^2 l \sum_i n_i^2 R_i^2}$$

(49)

The $\mathbf{n}=\mathbf{0}$ contribution is a cosmological constant which is divergent in this simple case, but irrelevant for the purpose of this review. For all $\mathbf{n}\neq\mathbf{0}$, the change of variables: $l' = \pi^2 l \sum_i n_i^2 R_i^2$ and integration over l' leads to

> $\mathbf{n}=\mathbf{0}$ 贡献是宇宙学常数，在这个简单情形下是发散的，但对于本综述的讨论而言无关紧要。对所有 $\mathbf{n}\neq\mathbf{0}$ ，做变量替换 $l' = \pi^2 l \sum_i n_i^2 R_i^2$ 并对 l' 积分后得到

$$V_{\text{eff}}(\phi)|_{\text{torus}} = -\sum_I (-)^{F_I} \frac{\Gamma\left(\frac{4+d}{2}\right)}{32\pi^{\frac{12+d}{2}}} \prod_{i=1}^{d} R_i \sum_{\mathbf{n}\neq\mathbf{0}} \frac{e^{2\pi i \sum_i n_i a_i^I(\phi)}}{\left[\sum_i n_i^2 R_i^2\right]^{\frac{4+d}{2}}} \tag{50}$$

which is finite for the ϕ -dependent part of the effective potential and, thus, computable in the field theory limit. It is important to stress again the importance, illustrated by this computation, to keep the whole infinite tower of KK modes and not to truncate them at some UV cutoff of the effective field theory.

> 该结果对有效势中依赖 ϕ 的部分是有限的，因此可以在场论极限下计算。需要再次强调: 这个计算清楚展示了，保留整个无限 KK 模式塔、而不是在有效场论的某个紫外截断处对其截断，是非常重要的。

The gauge field $\widehat{A}_M$ could in general be part of a non-abelian group. In this case, Wilson lines can be turned on along the Cartan generators leading to breaking patterns of the gauge group associated with vacuum expectation values in the adjoint representation.

> 规范场 $\widehat{A}_M$ 一般可以属于非阿贝尔群。这种情况下，开启沿嘉当生成元的威尔逊线，可以得到伴随表示真空期望值对应的规范群破缺模式。

Coordinate-Dependent Compactification and Supersymmetry Breaking

依赖坐标的紧化与超对称破缺

It may happen that in particular compactifications, the internal scalar component of the higher-dimensional gauge field does not survive as physical excitation in the spectrum, but still discrete values are allowed associated with a discrete symmetry of the theory. In this case, h_0 in (46) is absent and the parameter ω takes discrete values. Still, this leads to gauge group symmetry breaking, even in the absence of the corresponding Higgs scalars in the spectrum. The net effect can also be interpreted as discrete Wilson lines. This symmetry breaking by boundary conditions is also called ”coordinate dependent” (or Scherk-Schwarz) compactification [40,41], since zero modes of charged fields acquire z -internal coordinate dependence, as it can be seen in the KK-mode expansion (44).

在特殊的紧化过程中，高维规范场的内部标量分量可能不会作为物理激发保留在能谱中，但仍允许存在与理论的离散对称性关联的离散取值。这种情况下，(46) 式中的 h_0 不存在，参数 ω 取离散值。即便能谱中不存在对应的希格斯标量，这仍会引发规范群对称性破缺。其净效应也可以解释为离散威尔逊线。这种由边界条件引发的对称性破缺也被称为“依赖坐标的”(或谢尔克-施瓦茨) 紧化 [40,41]，因为带电场的零模会获得 z -内部坐标依赖，这一点可以从 KK 模展开式 (44) 中看出。

The above mechanism of gauge symmetry breaking can be generalized to any symmetry, such as supersymmetry that we discuss here. The relevant symmetry one can use to break supersymmetry is R-symmetry that acts as a phase rotation of the fermionic coordinates in the superspace. As a result, bosons and fermions which are different superfield components transform differently and have different boundary conditions (44), breaking supersymmetry. In general, such as in string theory, global continuous R-symmetry is broken by the compactification to a discrete subgroup, so that the supersymmetry breaking scale is not a free parameter, independent from the compactification, or the string scale [42].

上述规范对称性破缺机制可以推广到任意对称性，例如我们在此讨论的超对称。可用于破缺超对称的相关对称性是 R 对称性，它在超空间中对费米子坐标做相位转动。结果，属于不同超场分量的玻色子和费米子变换性质不同，满足不同的边界条件 (44)，从而破缺超对称。一般而言，例如在弦论中，整体连续 R 对称性会被紧化破缺为离散子群，因此超对称破缺尺度不是独立于紧化尺度或弦尺度的自由参数 [42]。

The simplest and universal example is a Z_2 parity that corresponds to the fermion number. Thus, fermions change sign and, thus, $q\omega = 1/2$ in (44) leading to KK modes with half-integer frequencies, corresponding to $a = 1/2$ in (45). On the other hand, bosons are invariant and their KK modes remain unaffected. The resulting supersymmetry breaking spectrum is identical to the Matsubara frequencies at finite temperature, where the compactified space coordinate z is replaced by time with the radius being replaced by the inverse temperature. Bosons are periodic with integer frequencies, while fermions are antiperiodic leading to half-integer frequencies [6, 43, 44] .

最简单且普适的例子是对应费米子数的 Z_2 宇称。因此费米子会改变符号，进而在 (44) 式中得到 $q\omega = 1/2$，产生具有半整数频率的卡鲁扎-克莱因模，对应 (45) 式中的 $a = 1/2$。另一方面，玻色子保持不变，它们的 KK 模不受影响。最终得到的超对称破缺能谱与有限温度下的松原频率一致，其中紧化空间坐标 z 被时间替换，半径被逆温度替换。玻色子具有整数频率，满足周期性，而费米子满足反周期性，得到半整数频率 [6, 43, 44]。

It turns out that coordinate-dependent compactifications are examples of freely acting orbifolds without fixed points, implying the absence of boundary states (associated with twisted sectors). This is illustrated in a simple Z_2 example described below.

事实上，依赖坐标的紧化是没有不动点的自由作用轨道泡利的例子，这意味着不存在 (与扭 sector 关联的) 边界态。下文的简单 Z_2 例子将说明这一点。

Freely acting orbifold Let us replace the interval S^1/Z_2 by combining the parity transformation $z \to -z$ with the Kaluza-Klein parity $(-)^n$. This amounts to shifting z by πR and eliminates the fixed points since the end-points of the interval are now exchanged by the orbifold action. Unlike the KK-mode decomposition (25) in the S^1/Z_2 orbifold, now both even and odd fields under z -parity can have cosine and sine expansions, depending on the KK-parity which is even for $n = 2k$ even integers and odd for $n = 2k - 1$ odd integers:

自由作用轨道泡利我们将区间 S^1/Z_2 替换为宇称变换 $z \to -z$ 与卡鲁扎-克莱因宇称 $(-)^n$ 的组合。这相当于将 z 平移 πR，并消除了不动点，因为区间的端点现在会被轨道泡利作用交换。与 S^1/Z_2 轨道泡利中的 KK 模分解 (25) 不同，现在对 z 宇称而言的偶场和奇场都可以展开为余弦和正弦形式，这取决于 KK 宇称: 当 $n = 2k$ 为偶数时 KK 宇称是偶，当 $n = 2k - 1$ 为奇数时 KK 宇称是奇:

$$\hat{\Phi}_{\text{even}}(x,z) = \frac{1}{\sqrt{\pi R}}\varphi_0^{e,e}(x) \tag{51}$$

$$+\sqrt{\frac{2}{\pi R}}\sum_{k=1}^{+\infty}\varphi_{2k}^{e,e}(x)\cos\frac{2kz}{R} + \sqrt{\frac{2}{\pi R}}\sum_{k=1}^{+\infty}\varphi_{2k-1}^{e,o}(x)\sin\frac{(2k-1)z}{R}$$

$$\hat{\Phi}_{\text{odd}}(x,z) = \sqrt{\frac{2}{\pi R}}\sum_{k=1}^{+\infty}\varphi_{2k-1}^{o,e}(x)\cos\frac{(2k-1)z}{R} + \sqrt{\frac{2}{\pi R}}\sum_{k=1}^{+\infty}\varphi_{2k}^{o,o}(x)\sin\frac{2kz}{R},$$

where the second superscript of the mode functions φ_n refers to the even (e) or odd (o) transformation with respect to the KK-parity that is combined with the action of the z -parity of the five-dimensional field and z -parity of the wave functions (even for cosine and odd for sin) to form an overall even action under Z_2 .

其中模函数 φ_n 的上标第二个指标指代相对于 KK 宇称的偶 ((e)) 或奇 (o) 变换，该变换与五维场的 z 宇称以及波函数的 z 宇称 (余弦为偶，正弦为奇) 组合后，在 Z_2 下整体为偶作用。

Scherk-Schwarz compactification Let us consider now a Scherk-Schwarz compactification described previously based on a Z_2 symmetry acting on the five-dimensional fields and imposing boundary conditions as in (44) with $q\omega = 1/2$, so that odd fields are antiperiodic. This leads to the following KK-mode expansions:

谢尔克-施瓦茨紧化现在我们讨论前文介绍的谢尔克-施瓦茨紧化，它基于作用在五维场上的 Z_2 对称性，并如 (44) 式在 $q\omega = 1/2$ 下施加边界条件，因此奇场满足反周期性。由此得到如下 KK 模展开:

$$\hat{\Phi}_P(x,z) = \frac{1}{\sqrt{2\pi R}}\sum_{n=-\infty}^{+\infty}\varphi_n^P(x)e^{in\frac{z}{R}} \tag{52}$$

$$= \frac{1}{\sqrt{2\pi R}}\varphi_0^P(x) + \frac{1}{\sqrt{\pi R}}\sum_{k=1}^{+\infty}\varphi_k^{P,R}(x)\cos\frac{2kz}{2R}$$

$$+\frac{1}{\sqrt{\pi R}}\sum_{k=1}^{+\infty}\varphi_k^{P,I}(x)\sin\frac{2kz}{2R}$$

$$\hat{\Phi}_A(x,z) = \frac{1}{\sqrt{2\pi R}}\sum_{n=-\infty}^{+\infty}\varphi_n^A(x)e^{i\left(n+\frac{1}{2}\right)\frac{z}{R}}$$

$$= \frac{1}{\sqrt{\pi R}}\sum_{k=1}^{+\infty}\varphi_k^{A,R}(x)\cos\frac{(2k-1)z}{2R} + \frac{1}{\sqrt{\pi R}}\sum_{k=1}^{+\infty}\varphi_k^{A,I}(x)\sin\frac{(2k-1)z}{2R},$$

where $P(A)$ refers to periodic (antiperiodic) fields, while the superscripts R and I stand for the real and imaginary part of the corresponding mode function.

其中 $P(A)$ 指代周期 (反周期) 场，上标 R 和 I 分别对应对应模函数的实部和虚部。

It is now easy to see that the expressions (51) and (52) can be identified upon doubling the radius from the interval in (51) to the circle in (52) and the identification:

现在很容易看出，若将半径加倍，从 (51) 式的区间变为 (52) 式的圆，就可以将表达式 (51) 和 (52) 等价起来，对应关系为:

$$\varphi_k^P = \varphi_{2k}^{e,e} - i\varphi_{2k}^{o,o} \; ; \; \varphi_k^A = \varphi_{2k}^{o,e} - i\varphi_{2k}^{e,o}. \tag{53}$$

Extra Dimensions and the Swampland Conjectures

额外维度与沼泽地猜想

Additional dimensions play a central role in string theory. Indeed, one does not have to live in the flat space-time of the critical string dimensions, but by means of a different ”dimensional reduction,” one can restrict the number of infinite dimensions of the space where the effective field theory lives. Several properties of the known string theory compactification models have recently been promoted as properties of all quantum theories of gravity by a series of conjectures. Two conjectures are of particular interest here. The first is the Distance Conjecture stating that in directions of large distances in the moduli space, there is a tower of light states with masses exponentially small with the proper distance measured in Planck units. The second is the Weak Gravity Conjecture stating that gravity is the weakest force. This implies the existence of a state with charge bigger than its mass in Planck units, allowing black holes to decay, evading stable remnants.

额外维度在弦论中占据核心地位。实际上，弦论并不要求我们生活在临界弦维度的平坦时空中，通过不同的「维数约化」，人们可以限制有效场论所处空间中无穷维维度的数量。近年来，已知弦论紧致化模型的若干性质被一系列猜想推广为所有量子引力理论都必须满足的性质。本文将重点关注其中两个猜想。第一个是距离猜想，该猜想指出: 在模空间中距离无穷远的方向上，存在一列光态塔，其质量随普朗克单位度量的固有距离呈指数衰减。第二个是弱引力猜想，该猜想指出引力是所有相互作用中最弱的力。这意味着必然存在一个电荷 (普朗克单位下) 大于其质量的态，从而允许黑洞衰变，避免形成稳定残余。

Very Weak Couplings

极弱耦合

The Swampland Conjectures [45,46] forbid arbitrarily small gauge coupling. This is if we can take it to zero, we get a global symmetry which is argued to be absent in quantum gravity. We will illustrate the obstruction to taking this limit in an example, that of the dark photon.

沼泽地猜想 [45,46] 禁止任意小的规范耦合。这是因为如果我们能将规范耦合取为零，就会得到一个整体对称性，而量子引力中被认为不存在整体对称性。我们将以暗光子为例说明取该极限的阻碍。

In string theory, one can suppress the strength of gauge coupling g_X by taking δ_X extra dimensions to be large. Indeed, one can express g_X as

在弦论中，我们可以通过让 δ_X 额外维度变大来压低 g_X 规范耦合的强度。事实上，我们可以将 g_X 表示为

$$g_X^2 = \frac{(2\pi)^{\delta_X+1} g_s}{V_X M_s^{\delta_X}} \tag{54}$$

or equivalently as

或等价表示为

$$g_X^2 = 2\pi g_s \left(\frac{8}{g_s^2}\right)^{\delta_X/d} \left(\frac{M_s}{M_{\rm Pl}}\right)^{2\delta_X/d} \tag{55}$$

Taking all the compact dimensions large ($\delta_X = d$) , we get [47, 48] :

当所有紧致维度都取大值 ($\delta_X = d$) 时，我们得到 [47, 48] :

$$g_X = \sqrt{\frac{16\pi}{g_s}\frac{M_s}{M_{\rm Pl}}} \sim 4 \times 10^{-14} \left(\frac{0.2}{g_s}\right)^{1/2} \left(\frac{M_s}{10\text{TeV}}\right) \tag{56}$$

For $g \sim 10^{-14}$, we see that new physics would appear at the scale: $\Lambda_{UV} \lesssim M_s \sim \; gM_{\rm Pl} \sim 10\text{TeV}$. This is the order of magnitude of the smallest string scale allowed by experiments, as we have seen above. So much small values of gauge couplings are not allowed in such theories. Moreover, independently of experiments, we see that the limit $g_X \to 0$ implies $\Lambda_{UV} \to 0$, and there is no more energy region where the effective theory is valid. This is an illustration of Magnetic Weak Gravity Conjecture.

对于 $g \sim 10^{-14}$ ， 我们可以看到新物理会出现在能标: $\Lambda_{UV} \lesssim M_s \sim \; gM_{\rm Pl} \sim 10\text{TeV}$ 。正如我们前文所述，这正是实验允许的最小弦标度的数量级。因此这类理论不允许规范耦合取过小的值。此外，不依赖实验我们也能看到，极限 $g_X \to 0$ 意味着 $\Lambda_{UV} \to 0$ ，不存在任何能让有效理论成立的能量区域了。这就是磁性弱引力猜想的一个实例。

On the other hand, taking $\delta_X = 0$, or alternatively chosing V_X of string size $\sim M_S^{-\delta_X}$, one obtains $g_X \sim \sqrt{g_s}$ while $M_s \sim g_s M_{\rm Pl}$ when all compactification sizes are of order the string length. Thus, g_X can because small by choosing a tiny string coupling g_s . A realization of this possibility is within the limit of little string theory [49, 50] , or using heterotic small instantons [51].

另一方面，如果取 $\delta_X = 0$ ，或者选择弦尺寸 $\sim M_S^{-\delta_X}$ 的 V_X ，当所有紧致化尺寸都为弦长量级时，可以得到 $g_X \sim \sqrt{g_s}$ ，同时 $M_s \sim g_s M_{\rm Pl}$ 。因此，通过选择极小的弦耦合 g_s 可以让 g_X 变小。这种可能性可以在小弦论 [49, 50] 的极限中实现，也可以通过杂化小瞬子实现 [51]。

Consider heterotic strings compactified on a $K3$ fibered over a two-dimensional base P^1 such that the approximation

考虑杂化弦紧致化在以二维底流形 P^1 为基的 $K3$ 纤维化上，满足近似条件

$$< V_{K3} V_{P^1} > \simeq < V_{K3} >< V_{P^1} > \tag{57}$$

is valid. Here, V_{P^1} and V_{K3} denote the volume of P^1 and $K3$, respectively. If all the compact dimensions are large

是成立的。此处 V_{P^1} 和 V_{K3} 分别表示 P^1 和 $K3$ 的体积。如果所有额外维度都很大

$$M_{\mathrm{Pl}}^2 = \frac{64\pi}{g_s^2} M_s^8 < V_{K3} V_{P1} > \tag{58}$$

We assume here that the Standard Model states arise from small instantons localized on $K3$. The volume V_{P^1} is then forced to remain of order one to avoid suppressing the Standard Model gauge couplings [51]. We have [48]:

我们此处假设标准模型态来自局域在 $K3$ 上的小瞬子。为了避免压低标准模型的规范耦合, 体积 V_{P^1} 必须保持在一阶量级 [51]。我们得到 [48]:

$$g_X^2 = \frac{g_s}{2} \frac{1}{M_s < V_{P^1} >} = 4\sqrt{\pi} \frac{M_s}{M_{\mathrm{Pl}}} M_s^2 \sqrt{< V_{K3} >} \sim 6 \times 10^{-14} \frac{M_s}{10\mathrm{TeV}} \tag{59}$$

where in the last estimate we used a string size $K3, < V_{K3} > \sim M_s^{-4}$. We can get now tiny gauge coupling for the dark photon not by large V_{K3} but also taking a tiny g_s . For instance, $g_s \sim 10^{-13}$ leads to $M_s \sim 10\mathrm{TeV}$.

其中最后一步估计我们使用了弦尺寸 $K3, < V_{K3} > \sim M_s^{-4}$ 。现在我们不需要通过增大 V_{K3} ，只需要取极小的 g_s 就可以得到暗光子的极小规范耦合。例如，$g_s \sim 10^{-13}$ 给出 $M_s \sim 10\mathrm{TeV}$ 。

We conclude that tiny couplings request either very large extra dimensions or a low string scale, therefore an infinite tower of KK states or of string resonances. The weaker the coupling, the lower the energy at which these towers appear.

我们得出结论: 极小耦合要么需要非常大的额外维度，要么需要低弦标度，因此会出现无穷的 KK 态塔或弦共振态塔。耦合越弱，这些态塔出现的能量就越低。

The Dark Dimension

暗维度

We very briefly mention the recent proposal for one extra dimension of length in the micrometer range. There are indications that if one continuously takes the limit of an AdS space cosmological constant toward zero, there is a tower of states that become very light with masses that go parametrically like $|\Lambda|^{-1/4}$ [52]. In [53], the authors assume that dS solutions will behave in this respect as AdS spaces. Further, they assume that these towers of states have the description of KK modes of an extra dimension. Given the tiny size of our Universe dark energy, these states are very light and do not decouple from the EFT:

我们非常简要地介绍近期提出的微米级额外维度方案。有迹象表明，如果连续取 AdS 空间宇宙学常数趋于零的极限，会存在一族变得极轻的态塔，其质量按参数化规律变化为 $|\Lambda|^{-1/4}$ [52]。文献 [53] 的作者假设，德西特解在这一方面的行为与 AdS 空间一致，还进一步假设这些态塔可以被描述为一个额外维度的 KK 模式。考虑到我们宇宙暗能量的极小取值，这些态质量极轻，不会从有效场论中退耦:

$$m \sim \lambda' \Lambda^{\frac{1}{4}}, \ \lambda' \sim 10 - 10^3 \tag{60}$$

This requires $R \sim 0.1 - 10\mu$m and corresponds to a fundamental scale of order $\sim 10^9 - 10^{10}$GeV which falls in the intermediate energies region mentioned above.

这要求 $R \sim 0.1 - 10\mu$m，对应的基本标度为 $\sim 10^9 - 10^{10}$GeV 量级，正好落在前文提到的中间能区范围内。

Cross-References

交叉引用

- A Lightning Introduction to String Theory

- 弦论快速导论

Effective Field Theory for Compact Binary Dynamics

致密双星动力学的有效场论

References

参考文献

1. T. Kaluza, Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.) 1921, 966-972 (1921). https://doi.org/10.1142/S021827 [arXiv:1803.08616 [physics.hist-ph]]
2. O. Klein, Z. Phys. 37, 895-906 (1926). https://doi.org/10.1007/BF01397481
3. O. Klein, Nature 118, 516 (1926). https://doi.org/10.1038/118516a0
4. A. Einstein, P. Bergmann, Ann. Math. 39, 683-701 (1938). https://doi.org/10.2307/1968642
5. E. Witten, [arXiv:1401.8048 [physics.hist-ph]]
6. I. Antoniadis, Phys. Lett. B 246, 377-384 (1990). https://doi.org/10.1016/0370- 2693(90)90617-F
7. I. Antoniadis, K. Benakli, Phys. Lett. B 326, 69-78 (1994). https://doi.org/10.1016/0370- 2693(94)91194-0, [arXiv:hep-th/9310151 [hep-th]]
8. I. Antoniadis, K. Benakli, M. Quiros, Phys. Lett. B 331, 313-320 (1994). https://doi.org/10.1016/0370-2693(94)91058-8, [arXiv:hep-ph/9403290 [hep-ph]]
9. N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali, Phys. Lett. B 429, 263-272 (1998). https://doi.org/10.1016/S0370-2693(98)00466-3, [arXiv:hep-ph/9803315 [hep-ph]]

10. I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali, Phys. Lett. B 436, 257-263 (1998). https://doi.org/10.1016/S0370-2693(98)00860-0, [arXiv:hep-ph/9804398 [hep-ph]]

11. K. Benakli, C. Branchina, G. Lafforgue-Marmet, [arXiv:2210.00477 [hep-th]]

12. K. Benakli, Phys. Rev. D 60, 104002 (1999). https://doi.org/10.1103/PhysRevD.60.104002, [arXiv:hep-ph/9809582 [hep-ph]]

13. C.P. Burgess, L.E. Ibanez, F. Quevedo, Phys. Lett. B 447, 257-265 (1999). https://doi.org/10.1016/S0370-2693(99)00006-4, [arXiv:hep-ph/9810535 [hep-ph]]

14. E.G. Adelberger, B.R. Heckel, A.E. Nelson, Ann. Rev. Nucl. Part. Sci. 53, 77-121 (2003). https://doi.org/10.1146/annure [arXiv:hep-ph/0307284 [hep-ph]]

15. D.J. Kapner, T.S. Cook, E.G. Adelberger, J.H. Gundlach, B.R. Heckel, C.D. Hoyle, H.E. Swanson, Phys. Rev. Lett. 98, 021101 (2007). https://doi.org/10.1103/PhysRevLett.98.021101, [arXiv:hep-ph/0611184 [hep-ph]]

16. W.H. Tan, A.B. Du, W.C. Dong, S.Q. Yang, C.G. Shao, S.G. Guan, Q.L. Wang, B.F. Zhan, P.S. Luo, L.C. Tu et al., Phys. Rev. Lett. 124(5), 051301 (2020). https://doi.org/10.1103/ PhysRevLett.124.051301

17. N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali, Phys. Rev. D 59, 086004 (1999). https://doi.org/10.1103/PhysRevD.59.08 [arXiv:hep-ph/9807344 [hep-ph]]

18. K. Benakli, S. Davidson, Phys. Rev. D 60, 025004 (1999). https://doi.org/10.1103/PhysRevD.60.025004, [arXiv:hep-ph/9810280 [hep-ph]]

19. L.J. Hall, D. Tucker-Smith, Phys. Rev. D 60, 085008 (1999). https://doi.org/10.1103/ PhysRevD.60.085008, [arXiv:hep-ph/9904267 [hep-ph]]

20. C. Hanhart, J.A. Pons, D.R. Phillips, S. Reddy, Phys. Lett. B 509, 1-9 (2001). https://doi.org/ 10.1016/S0370-2693(01)00544-5, [arXiv:astro-ph/0102063 [astro-ph]]

21. S. Hannestad, G.G. Raffelt, Phys. Rev. D 67, 125008 (2003); [Erratum: Phys. Rev. D 69, 029901 (2004)]. https://doi.org/10.1103/PhysRevD.69.029901, [arXiv:hep-ph/0304029 [hep-ph]]

22. I. Antoniadis, K. Benakli, M. Quiros, Phys. Lett. B 460, 176-183 (1999). https://doi.org/10.1016/S0370-2693(99)00764-9, [arXiv:hep-ph/9905311 [hep-ph]]

23. G.F. Giudice, R. Rattazzi, J.D. Wells, Nucl. Phys. B 544, 3-38 (1999). https://doi.org/10.1016/ S0550-3213(99)00044-9, [arXiv:hep-ph/9811291 [hep-ph]]

24. E.A. Mirabelli, M. Perelstein, M.E. Peskin, Phys. Rev. Lett. 82, 2236-2239 (1999). https://doi.org/10.1103/PhysRevLett.8 [arXiv:hep-ph/9811337 [hep-ph]]

25. T. Han, J.D. Lykken, R.J. Zhang, Phys. Rev. D 59, 105006 (1999). https://doi.org/10.1103/ PhysRevD.59.105006, [arXiv:hep-ph/9811350 [hep-ph]]

26. J.L. Hewett, Phys. Rev. Lett. 82, 4765-4768 (1999). https://doi.org/10.1103/PhysRevLett.82.4765, [arXiv:hep-ph/9811356 [hep-ph]]

27. R.L. Workman et al., [Particle Data Group], PTEP 2022, 083C01 (2022). https://doi.org/10.1093/ptep/ptac097

28. G. Aad et al., [ATLAS], Phys. Rev. D 103(11), 112006 (2021). https://doi.org/10.1103/ PhysRevD.103.112006, [arXiv:2102.10874 [hep-ex]]

29. A. Tumasyan et al., [CMS], JHEP 11, 153 (2021). https://doi.org/10.1007/JHEP11(2021)153, [arXiv:2107.13021 [hep-ex]]

30. G. Aad et al., [ATLAS], JHEP 03, 145 (2020). https://doi.org/10.1007/JHEP03(2020)145, [arXiv:1910.08447 [hep-ex]]

31. A.M. Sirunyan et al., [CMS], JHEP 05, 033 (2020). https://doi.org/10.1007/JHEP05(2020)033, [arXiv:1911.03947 [hep-ex]]

32. A.M. Sirunyan et al., [CMS], Eur. Phys. J. C 78(9), 789 (2018); [Erratum: Eur. Phys. J. C 82(4), 379 (2022)]. https://doi.org/10.1140/epjc/s10052-018-6242-x, [arXiv:1803.08030 [hep-ex]]

33. E. Accomando, I. Antoniadis, K. Benakli, Nucl. Phys. B 579, 3-16 (2000). https://doi.org/10.1016/S0550-3213(00)00123-1, [arXiv:hep-ph/9912287 [hep-ph]]

34. S. Cullen, M. Perelstein, M.E. Peskin, Phys. Rev. D 62, 055012 (2000). https://doi.org/10.1103/PhysRevD.62.055012, [arXiv:hep-ph/0001166 [hep-ph]]

35. I. Antoniadis, K. Benakli, A. Laugier, JHEP 05, 044 (2001). https://doi.org/10.1088/1126-6708/2001/05/044, [arXiv:hep-th/0011281 [hep-th]]

36. G. Aad et al., [ATLAS], Phys. Rev. D 100(5), 052013 (2019). https://doi.org/10.1103/PhysRevD.100.052013, [arXiv:1906.05609 [hep-ex]]

37. A.M. Sirunyan et al., [CMS], JHEP 06, 128 (2018). https://doi.org/10.1007/JHEP06(2018)128, [arXiv:1803.11133 [hep-ex]]

38. I. Antoniadis, K. Benakli, M. Quiros, New J. Phys. 3, 20 (2001). https://doi.org/10.1088/1367-2630/3/1/320, [arXiv:hep-th/0108005 [hep-th]]

39. K. Benakli, C. R. Phys. 8, 1048-1057 (2007). https://doi.org/10.1016/j.crhy.2007.02.001

40. J. Scherk, J.H. Schwarz, Phys. Lett. B 82, 60-64 (1979). https://doi.org/10.1016/0370-2693(79)90425-8

41. J. Scherk, J.H. Schwarz, Nucl. Phys. B 153, 61-88 (1979). https://doi.org/10.1016/0550-3213(79)90592-3

42. I. Antoniadis, C. Bachas, D.C. Lewellen, T.N. Tomaras, Phys. Lett. B 207, 441-446 (1988). https://doi.org/10.1016/0370-2693(88)90679-X

43. R. Rohm, Nucl. Phys. B 237, 553-572 (1984). https://doi.org/10.1016/0550-3213(84)90007-5

44. C. Kounnas, M. Porrati, Nucl. Phys. B 310, 355-370 (1988). https://doi.org/10.1016/0550-3213(88)90153-8

45. C. Vafa, [arXiv:hep-th/0509212 [hep-th]]

46. N. Arkani-Hamed, L. Motl, A. Nicolis, C. Vafa, JHEP 06, 060 (2007). https://doi.org/10.1088/1126-6708/2007/06/060, [arXiv:hep-th/0601001 [hep-th]]

47. K. Benakli, C. Branchina, G. Lafforgue-Marmet, Eur. Phys. J. C 80(12), 1118 (2020). https://doi.org/10.1140/epjc/s10052-020-08691-4, [arXiv:2007.02655 [hep-ph]]

48. L.A. Anchordoqui, I. Antoniadis, K. Benakli, D. Lust, Phys. Lett. B 810, 135838 (2020). https://doi.org/10.1016/j.physle [arXiv:2007.11697 [hep-th]]

49. I. Antoniadis, S. Dimopoulos, A. Giveon, JHEP 05, 055 (2001). https://doi.org/10.1088/1126-6708/2001/05/055, [arXiv:hep-th/0103033 [hep-th]]

50. I. Antoniadis, A. Arvanitaki, S. Dimopoulos, A. Giveon, Phys. Rev. Lett. 108, 081602 (2012). https://doi.org/10.1103/PhysRevLett.108.081602, [arXiv:1102.4043 [hep-ph]]

51. K. Benakli, Y. Oz, Phys. Lett. B 472, 83-88 (2000). https://doi.org/10.1016/S0370-2693(99)01422-7, [arXiv:hep-th/9910090 [hep-th]]

52. D. Lüst, E. Palti, C. Vafa, Phys. Lett. B 797, 134867 (2019). https://doi.org/10.1016/j.physletb.2019.134867, [arXiv:1906.05225 [hep-th]]

53. M. Montero, C. Vafa, I. Valenzuela, [arXiv:2205.12293 [hep-th]]